Robust and Nonparametric Statistical Tools for Big Data in Neuroscience

UNIVERSIDAD EAFIT

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Seminar
PhD in Mathematical Engineering

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Motivation: fMRI problem

Figure: **Functional Magnetic Resonance**

Motivation: fMRI problem

Figure: **Stimulus**

Motivation: fMRI problem

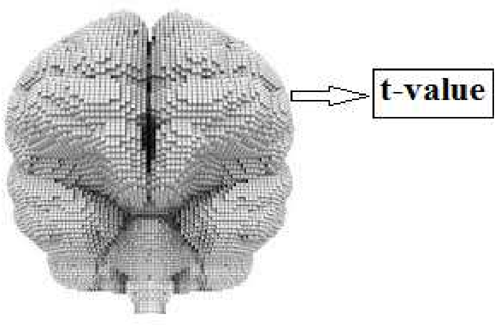

Figure: **900000 Voxels**

Motivation: fMRI problem

The theoretical activity is compared with that observed in each voxel.

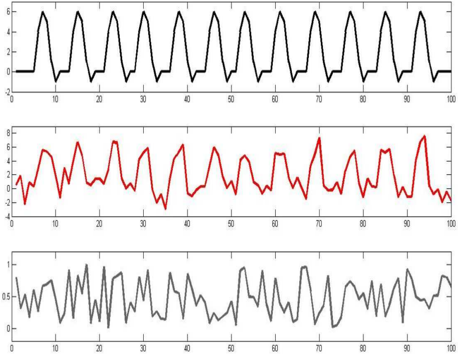

Figure: **Activation and non-activation**

Motivation: fMRI problem

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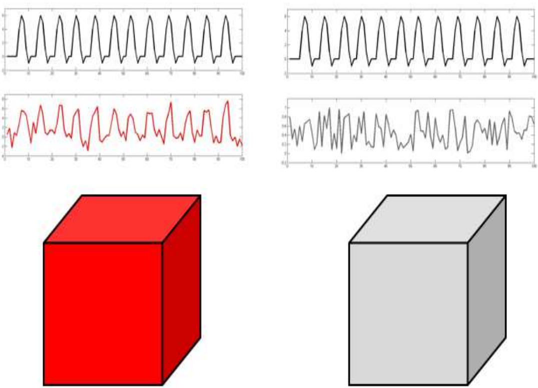
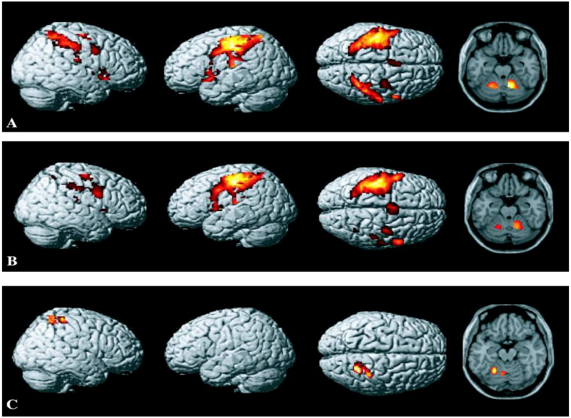


Figure: **Activation and non-activation**

Motivation: fMRI problem



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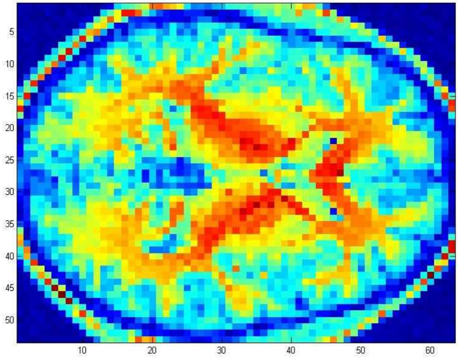

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Motivation: fMRI problem

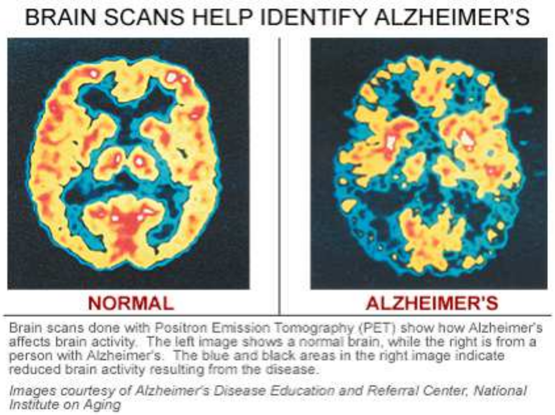

Figure: **Identification**

Main Goal

Find robust and non parametric estimators to explain linear dependence and apply them in the construction of covariance matrices to improve the significance of linear multivariate statistical models in the presence of outliers

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Hypothesis 1

Significance in linear models is too sensitive in the presence of outliers.

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Hypothesis 1

Significance in linear models is too sensitive in the presence of outliers.

Hypothesis 2

Significance in linear models can be improved with robust estimators or non parametric estimators

Outline

1. **Robustness: a fast review**
2. **General Lineal Model (GLM)**
3. **Robust GLM, with an explanatory variables**
4. **Robust GLM, multivariate case**
5. **Non-parametric estimators**
6. **Conclusions**
7. **Research lines**

Population: unknown parameters. $\theta = \int$

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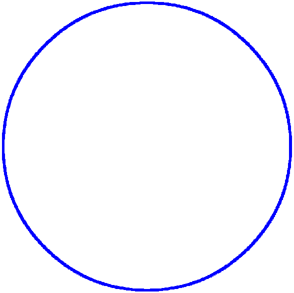

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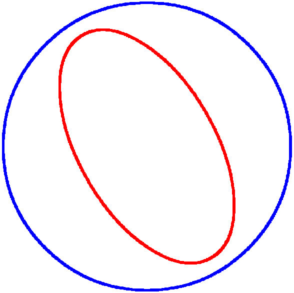
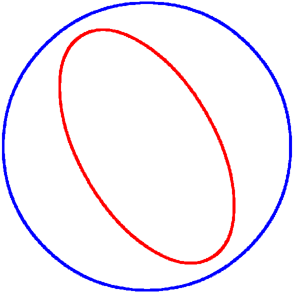


Sample: estimators. $\hat{\theta} = \sum$

Population: unknown parameters. $\theta = \int$



Sample: estimators. $\hat{\theta} = \sum$

$\mu = \int f(x)dx$ **the estimator** $\bar{X} = \frac{\sum x_i}{n}$

Example

Table: Simulated data from $N(\color{red}{0}, 1)$

0.354	-0.838	0.1204	-0.353	-0.3853	2.1261	-0.268
-0.16	-1.038	-0.273	0.7939	0.45153	0.858	0.6697
-1.838	2.5976	-0.282	0.1757	-0.5986	0.9407	-0.036
-0.531	0.6208	1.4303	-1.044	-0.1349	0.2476	-0.397
-1.432	1.0849	-1.12	0.6788	-1.1078	-0.245	0.5031
-1.136	-0.011	1.5668	0.6654	-0.6194	-1.481	-0.446
2.0077	-0.898	-0.886	-2.145	-2.435	-0.396	-0.116

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$$\bar{X} = \frac{\sum_{i=1}^{49} x_i}{49} = -0.09$$

Example

Table: Data from $N(0, 1)$. Contaminated data

0.354	-0.838	0.1204	-0.353	-0.3853	2.1261	-0.268
-0.16	-1.038	-0.273	8.2356	0.45153	0.858	0.6697
-1.838	2.5976	-0.282	0.1757	-0.5986	0.9407	12.266
-3.82	0.6208	1.4303	-1.044	-0.1349	0.2476	-0.397
-1.432	1.0849	-1.12	0.6788	-1.1078	-0.245	0.5031
-1.136	-0.011	1.5668	0.6654	-0.6194	-1.481	-0.446
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$$\bar{X} = \frac{\sum_{i=1}^{49} x_i}{49} = 0.4252$$

Example

Table: Data from $N(0, 1)$. Contaminated data

-5.132	-3.82	-2.1448	-1.4809	-1.4318	-1.14	-1.1199
-1.108	-1.044	-1.0384	-0.8979	-0.886	-0.84	-0.6194
-0.599	-0.531	-0.4462	-0.3974	-0.3956	-0.39	-0.2819
-0.273	-0.268	-0.2455	-0.1604	-0.1349	-0.01	0.1204
0.176	0.2476	0.354	0.45153	0.5031	0.621	0.6654
0.67	0.6788	0.7939	0.85803	0.9407	1.085	1.4303
1.567	2.0077	2.1261	2.59761	8.2356	9.266	12.266

$$\bar{X} = \frac{\sum_{i=1}^{49} x_i}{49} = 0.4252$$

Example

Table: Data from $N(0, 1)$. Contaminated data

X	X	X	-1.4809	-1.4318	-1.14	-1.1199
-1.108	-1.044	-1.0384	-0.8979	-0.886	-0.84	-0.6194
-0.599	-0.531	-0.4462	-0.3974	-0.3956	-0.39	-0.2819
-0.273	-0.268	-0.2455	-0.1604	-0.1349	-0.01	0.1204
0.176	0.2476	0.354	0.45153	0.5031	0.621	0.6654
0.67	0.6788	0.7939	0.85803	0.9407	1.085	1.4303
1.567	2.0077	2.1261	2.59761	X	X	X

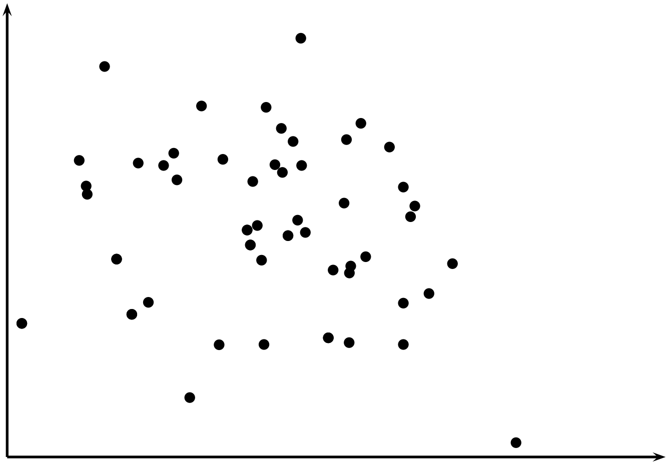
Trimmed mean

$$\bar{X}_\alpha = \frac{\sum_{i=k_1}^{k_2} x_{[i]}}{k_2 - k_1 + 1}$$

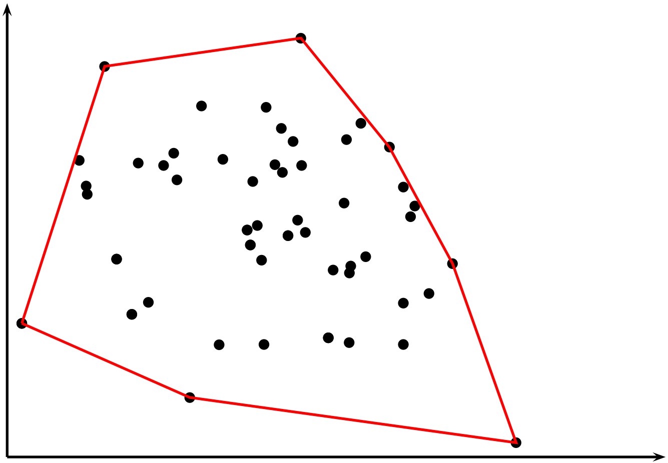
$k_1 = [\frac{\alpha}{2} n] \quad k_2 = [n(1 - \frac{\alpha}{2})]$

$$\bar{X}_{0.1} = 0.0503$$

Depth



Depth



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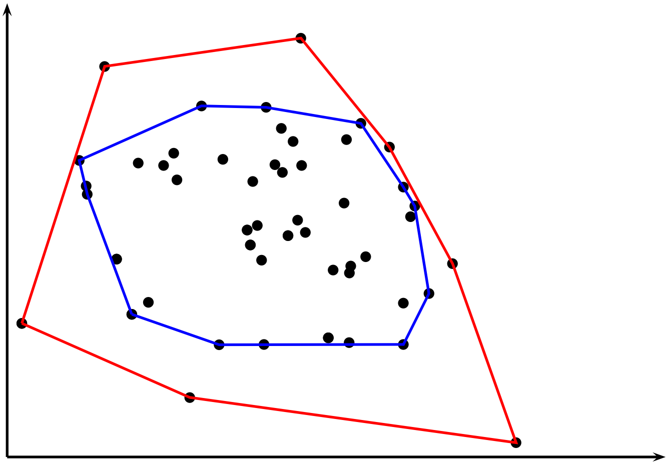

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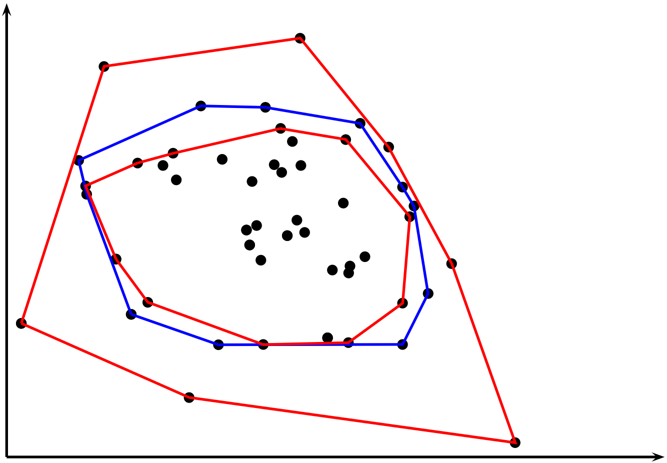

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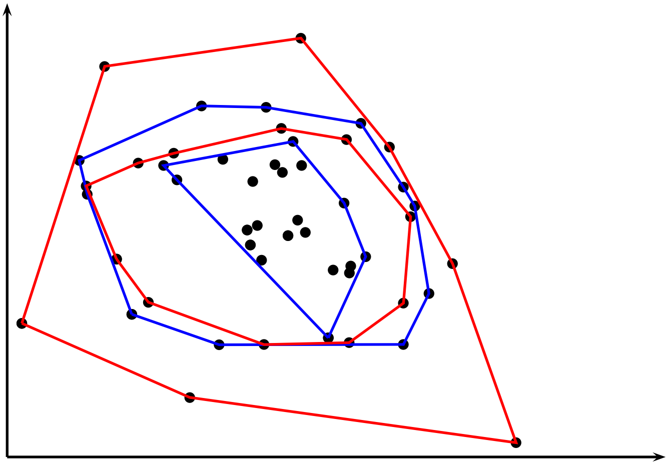

Dependent uniform distribution in dimension 2

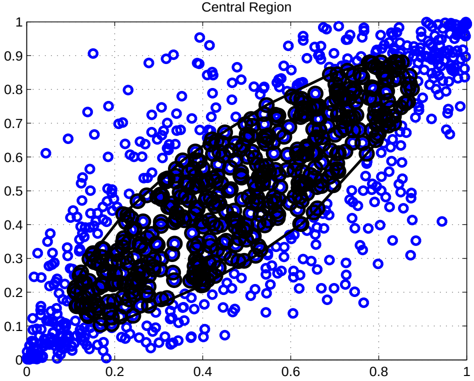

Figure: Random Tukey Depth

Normal distribution in dimension 3

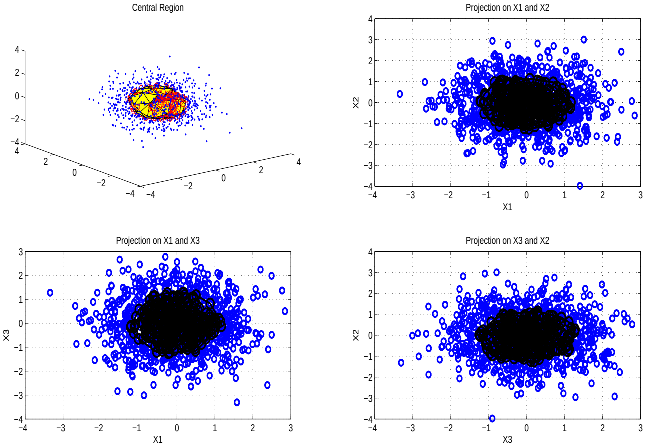

Figure: Random Tukey Depth

Central Region: univariate case

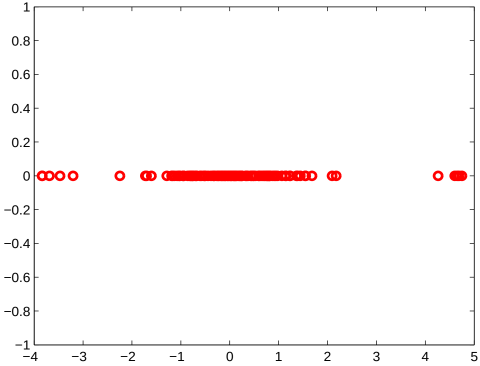

Figure: Data

Central Region: univariate case

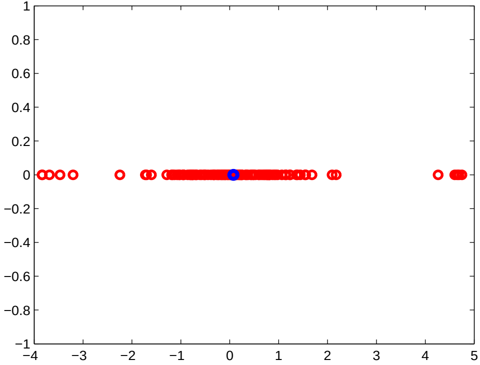

Figure: **Median**

Central Region: univariate case

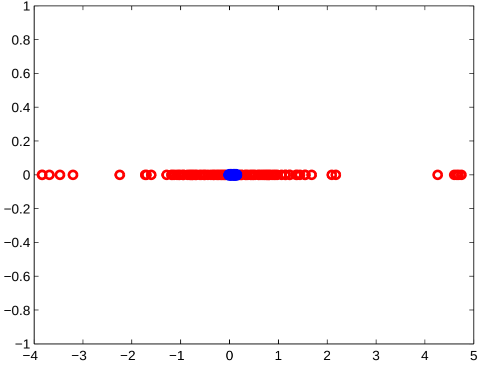

Figure: **Central Region** 10%

Central Region: univariate case

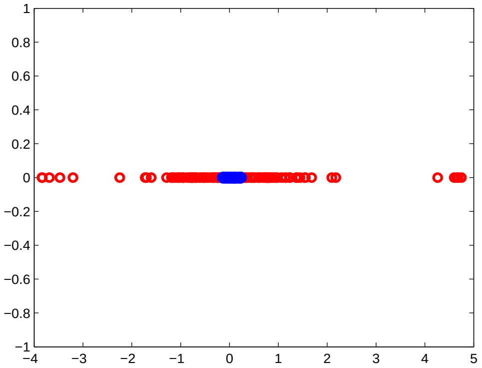

Figure: **Central Region** 20%

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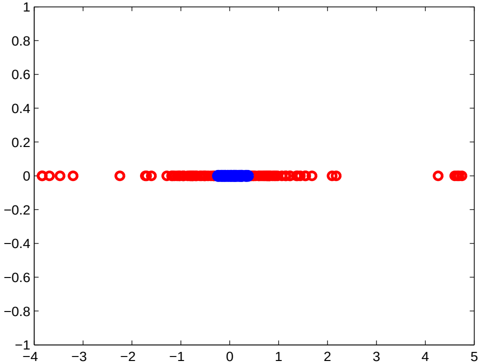


Figure: **Central Region** 30%

Central Region: univariate case

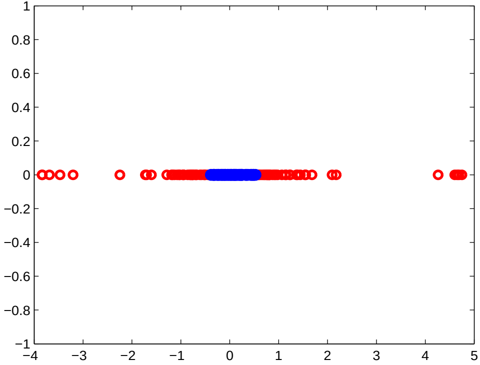


Figure: **Central Region** 40%

Central Region: univariate case

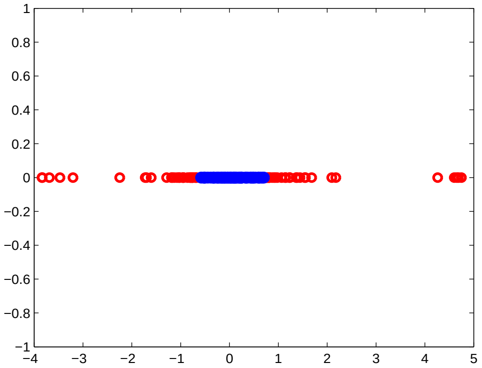


Figure: **Central Region** 50%

Central Region: univariate case

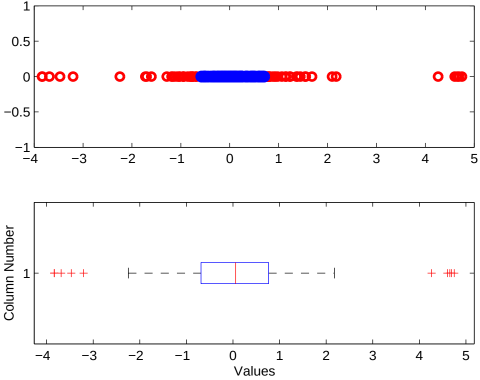


Figure: **Box plot**

Central Region: bivariate case

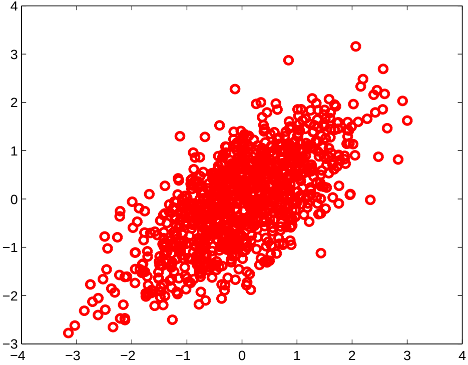


Figure: Data

Central Region: bivariate case

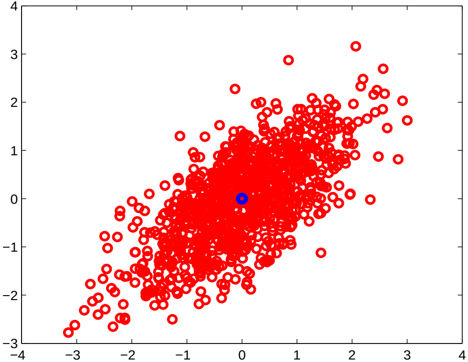


Figure: **deepest point**

Central Region: bivariate case

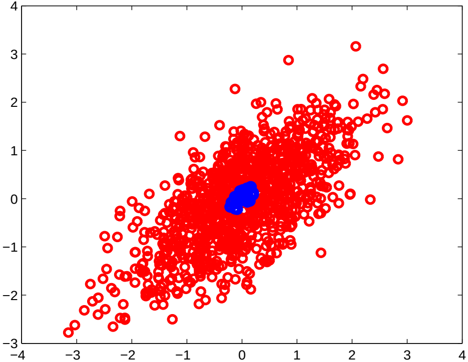


Figure: **Central Region** 10%

Central Region: bivariate case

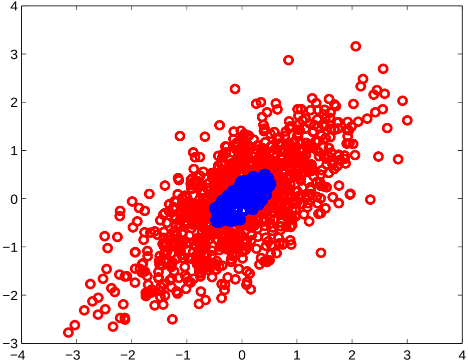


Figure: **Central Region** 30%

Central Region: bivariate case

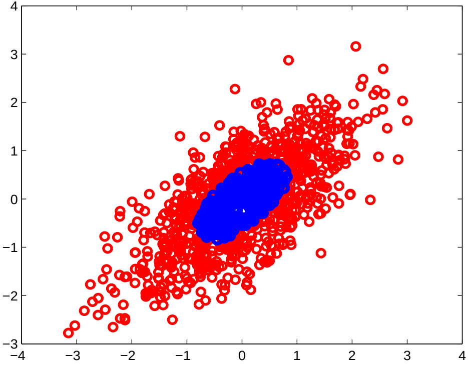


Figure: **Central Region** 40%

Central Region: bivariate case

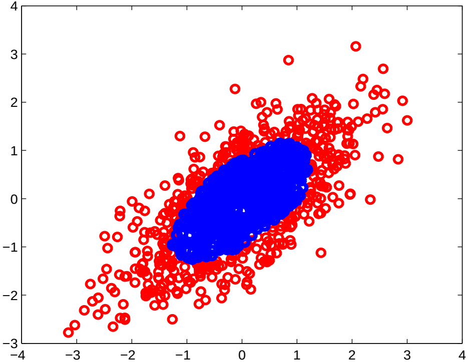


Figure: **Central Region** 50%

Central Region: functional case

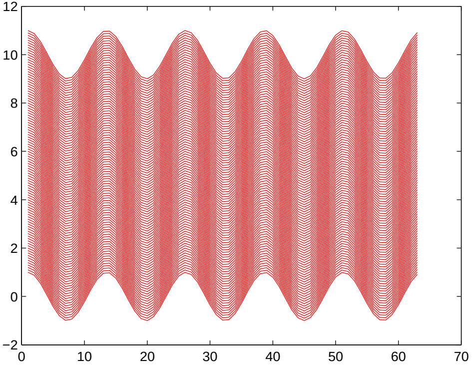


Figure: Data

Central Region: functional case

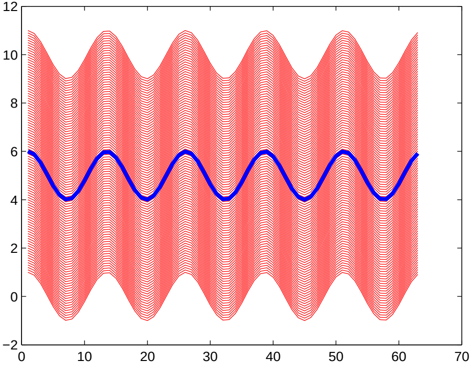


Figure: **deepest curve**

Central Region: functional case

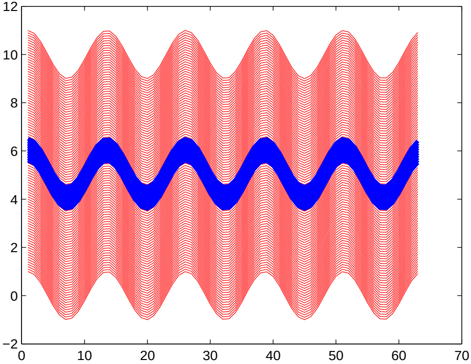


Figure: **Central Region** 10%

Central Region: functional case

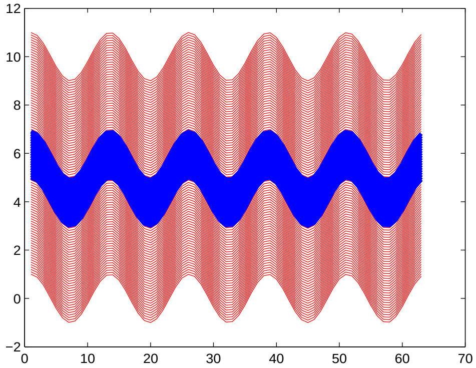


Figure: **Central Region** 20%

Central Region: functional case

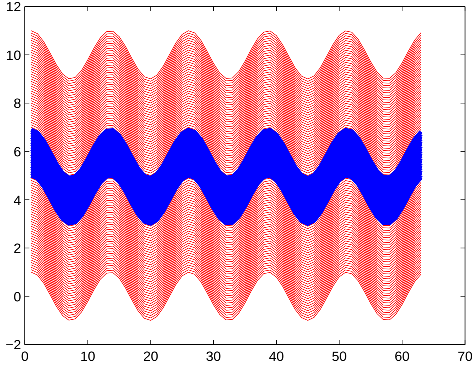


Figure: **Central Region** 20%

Central Region: functional case

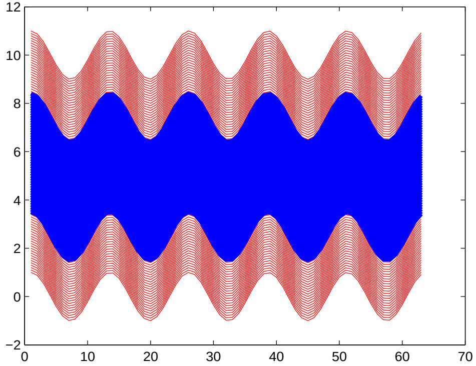


Figure: **Central Region** 50%

Central Region: functional case

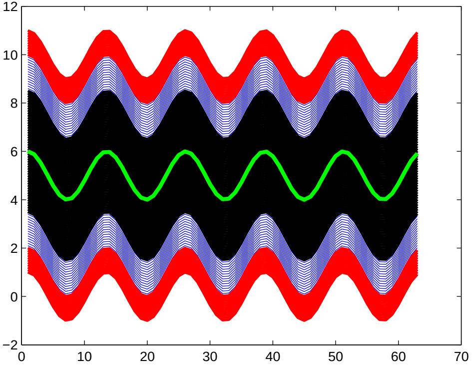


Figure: **Central Region, Extremes**

Central Region: functional case

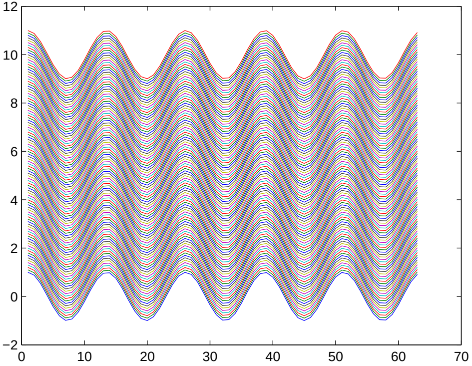


Figure: Data

Central Region: functional case

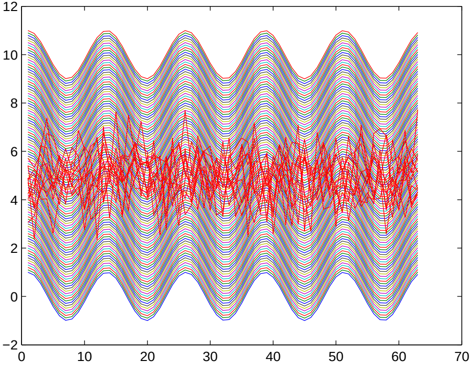


Figure: Contaminated Data

Central Region: functional case

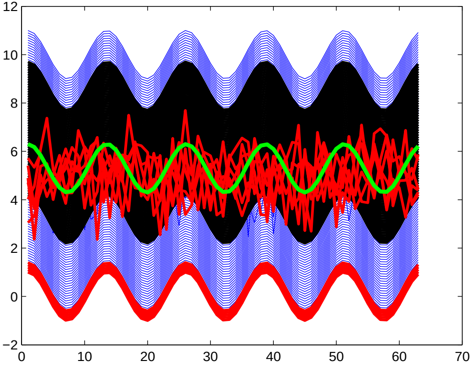
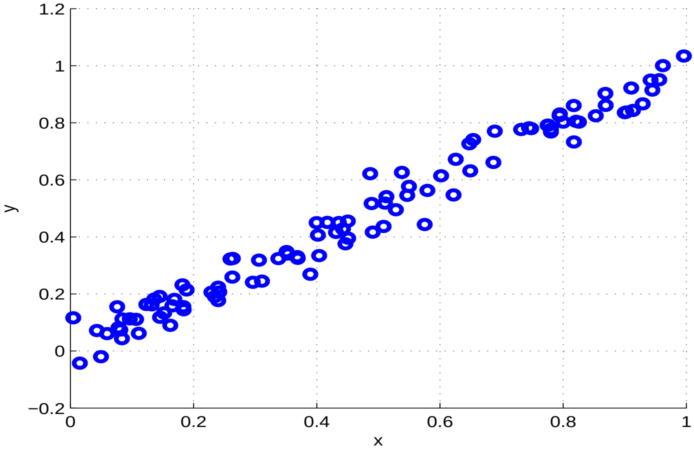


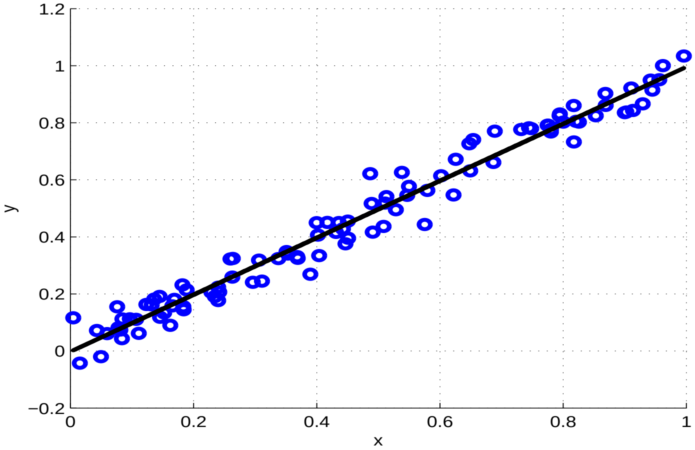
Figure: **Central Region, Extremes**

- Robustness:a fast review
- Central Region: bivariate case
- Central Region: functional case
- **General Linear Model (GLM)**
- Robust GLM, with an explanatory variable
  - Contaminating a 5%
  - Contaminating a 10%
- Robust GLM, Multiavarite case
  - Contaminating a 5%
  - Contaminating a 10%
- Non parametric estimators
- Conclusions
- Research Lines

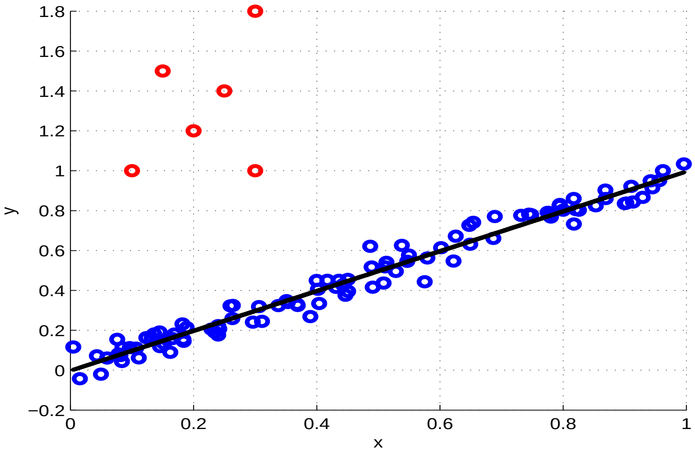
General Linear Model



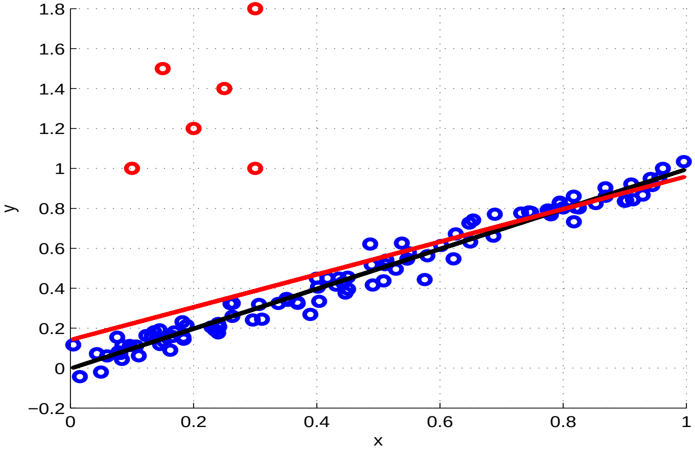
General Linear Model



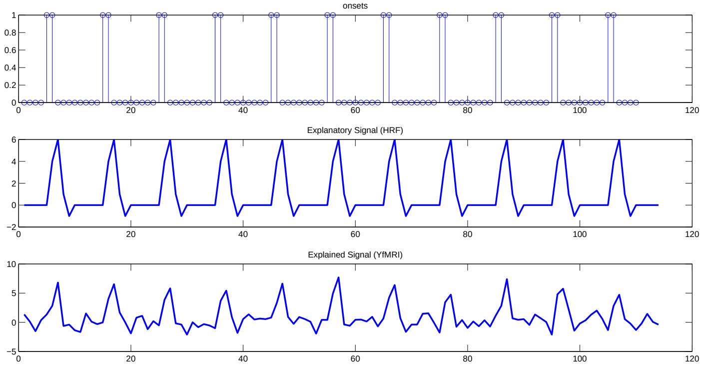
General Linear Model



General Linear Model

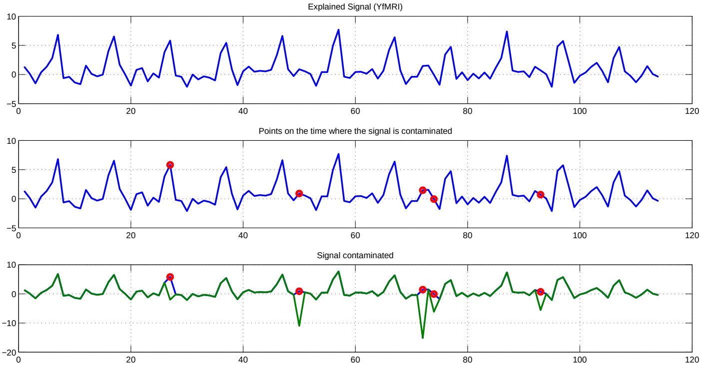


Significant variable



$$\mathbf{Y} = \mathbf{X} + N(0,1)$$

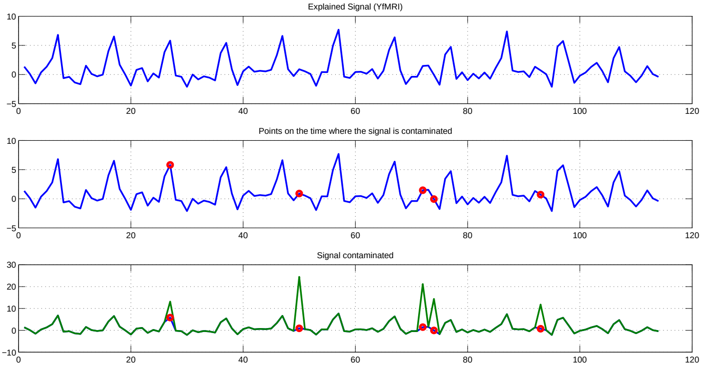
Significant variable and Contaminating a 5%



The red points will be generated by:

- **Normal**$(0, \sigma^2)$, σ^2 **varying from 1 to 100.**

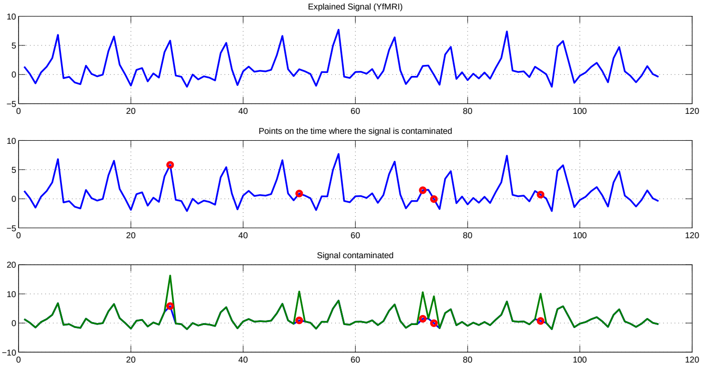
Significant variable and Contaminating a 5%



The red points will be generated by:

- **Normal**(μ, σ^2), **both varying from 1 to 100.**

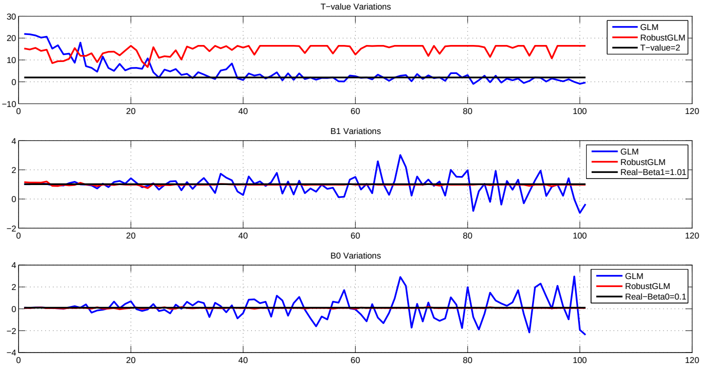
Significant variable and Contaminating a 5%



The red points will be generated by:

- **Normal**$(\mu, 1)$, μ **varying from 1 to 100.**

Significant variable and Contaminating a 5%



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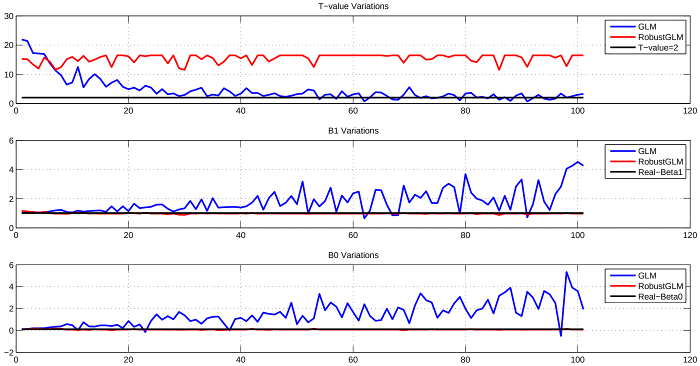

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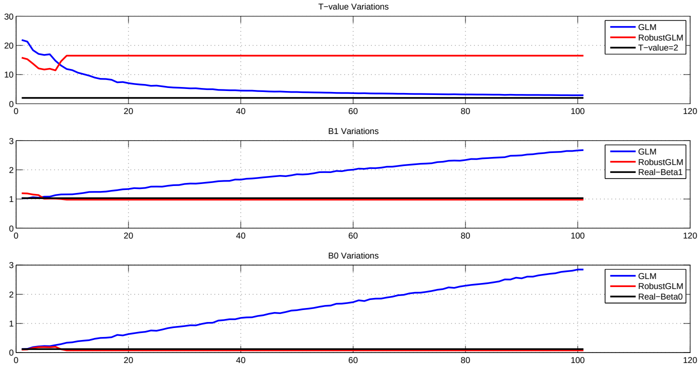

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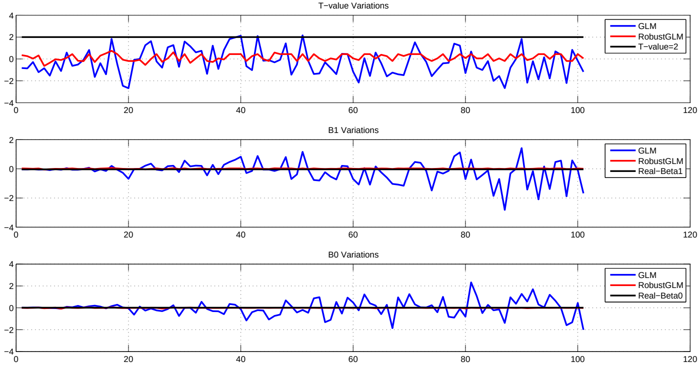

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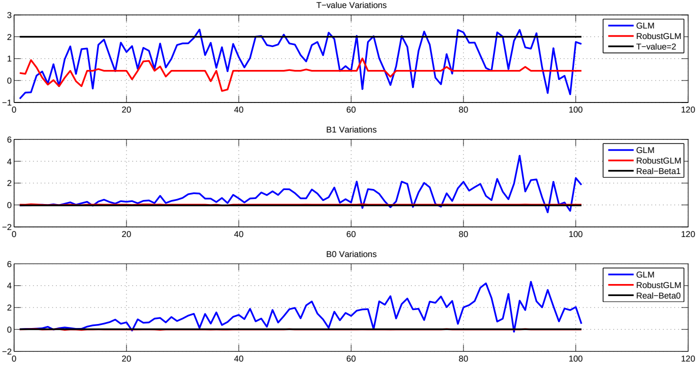

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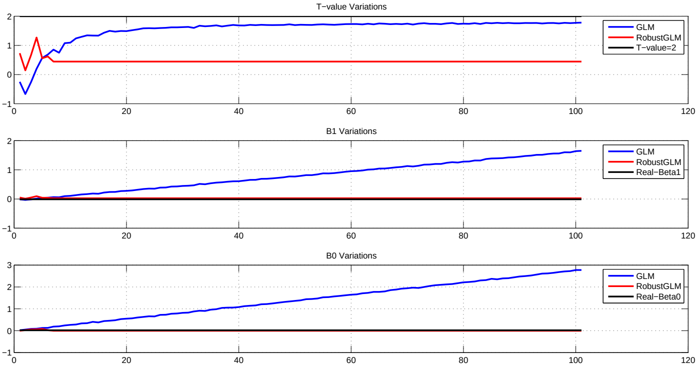

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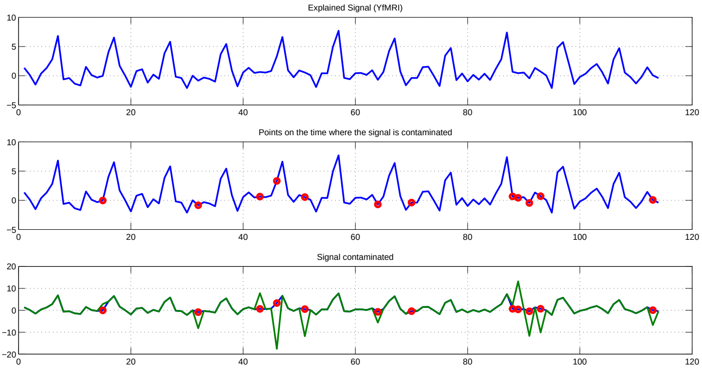

The red points will be generated by:

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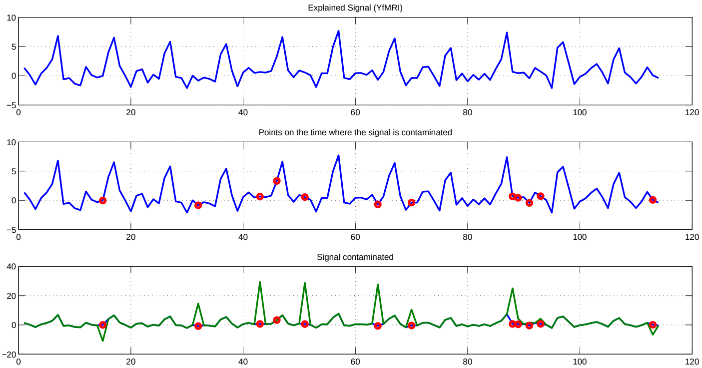

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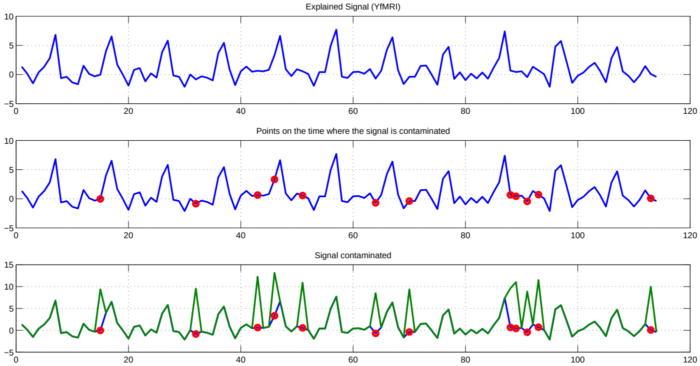

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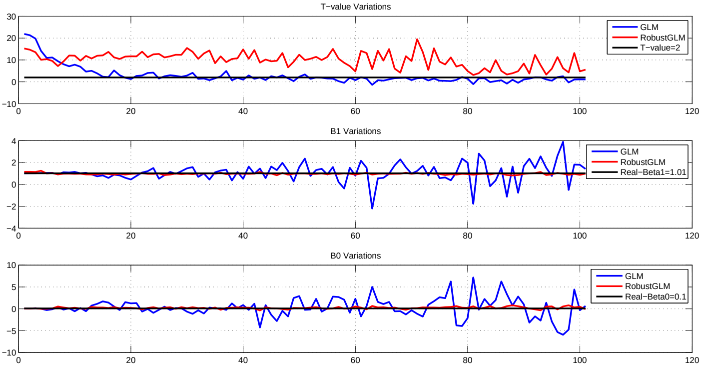

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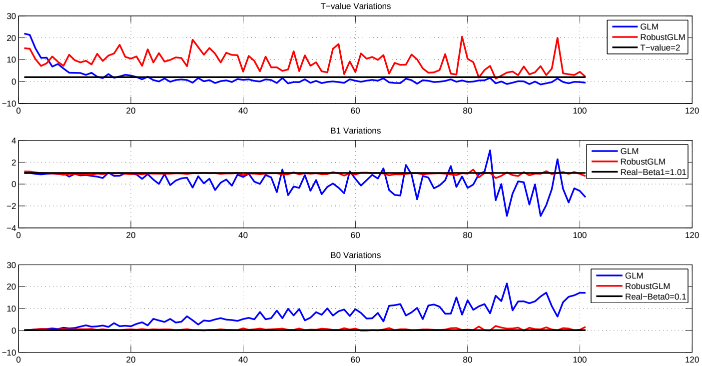

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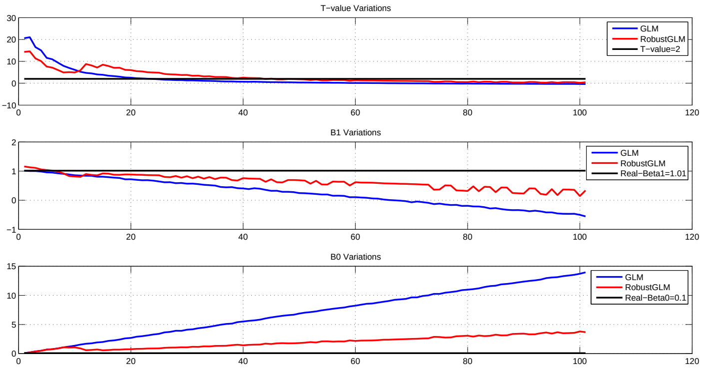

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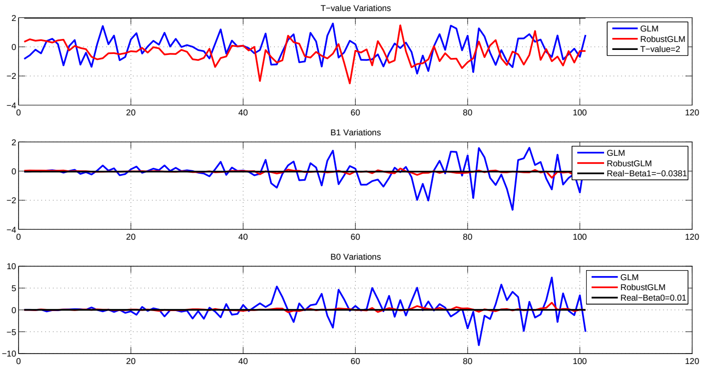

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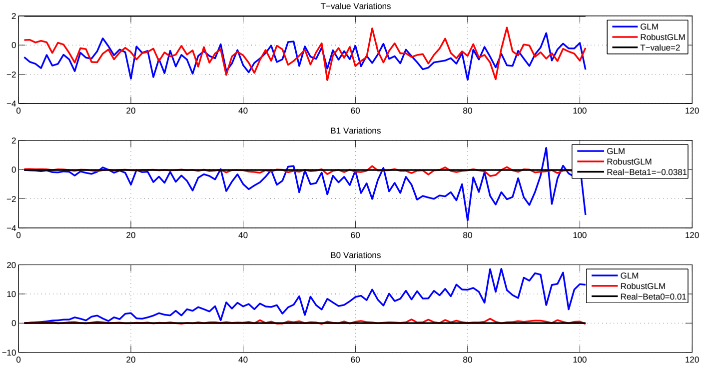

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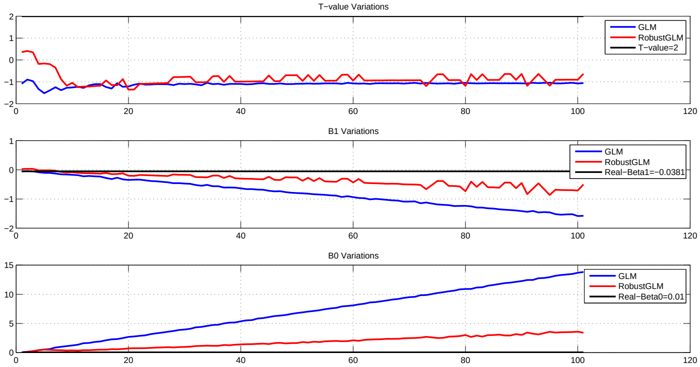

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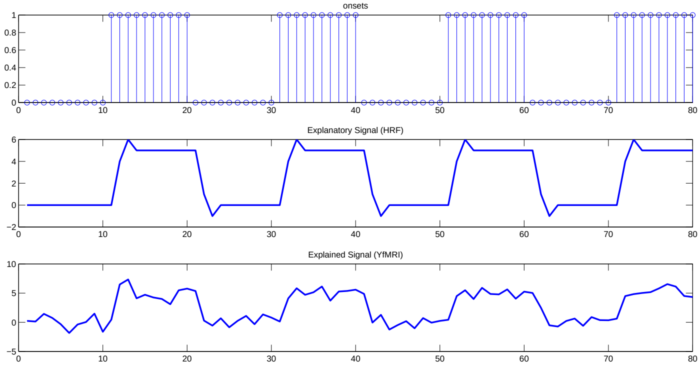
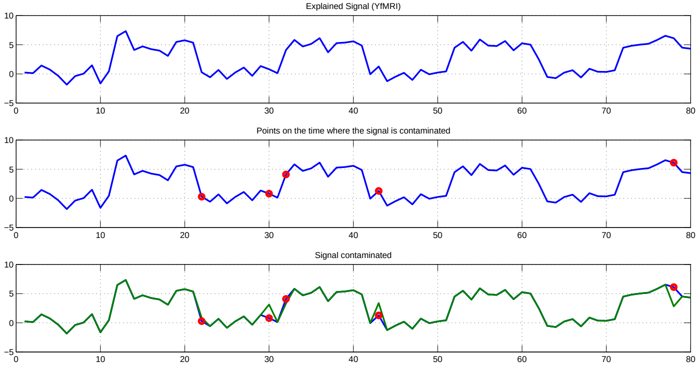
Significant variable



$$\mathbf{Y} = X_1 + \beta_2 X_2 + \cdots + \beta_6 X_6 + N(0,1)$$

Significant variable and Contaminating a 5%



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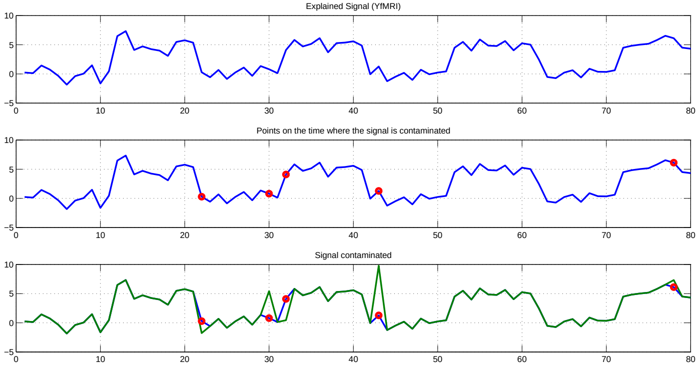

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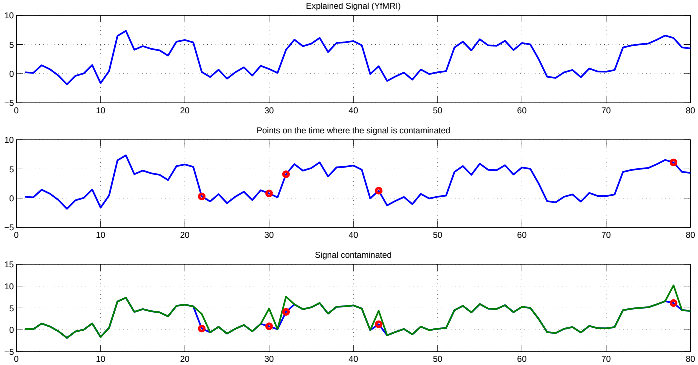

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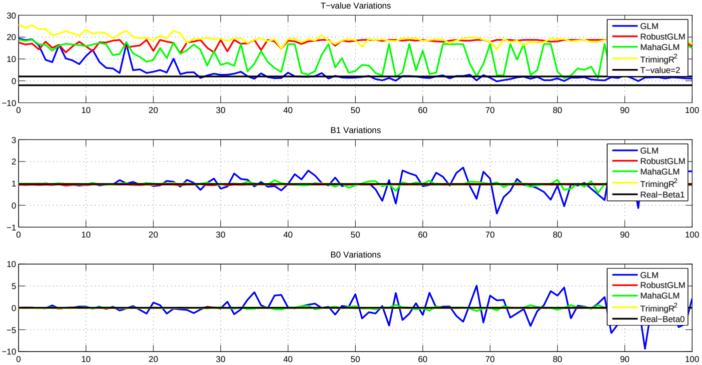

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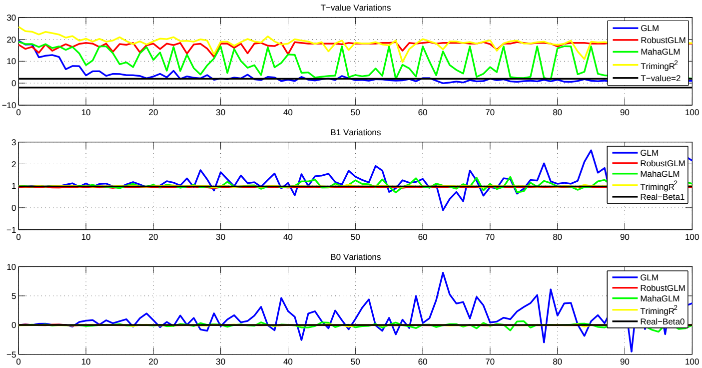

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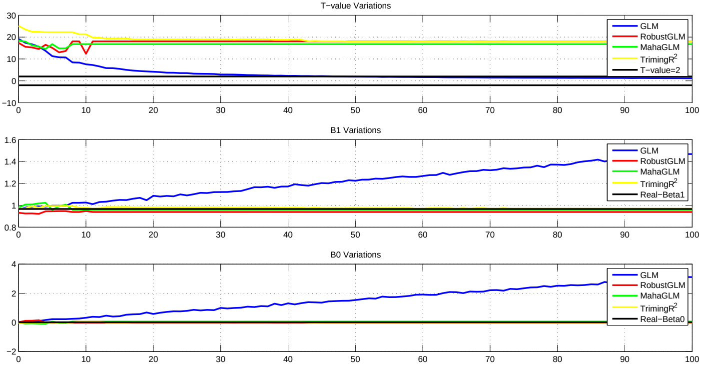

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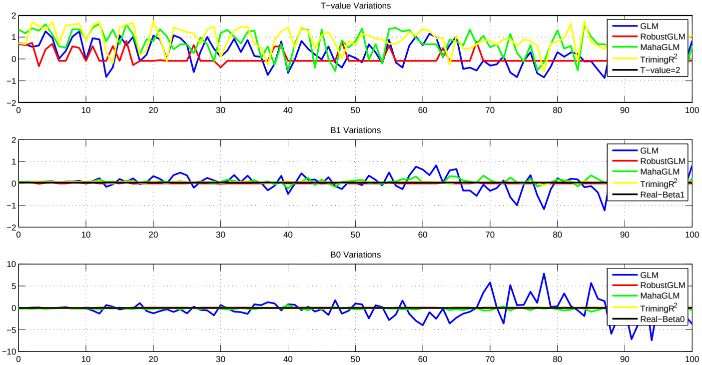

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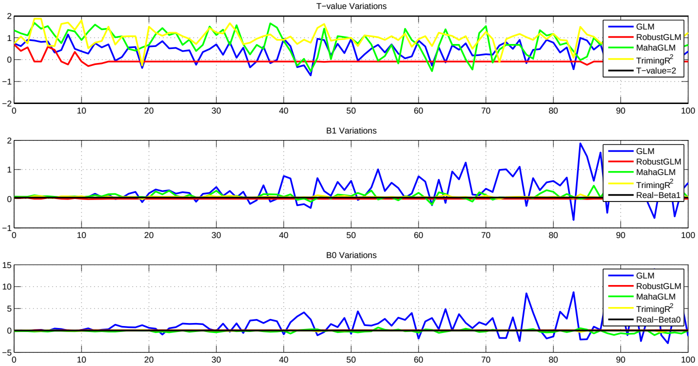

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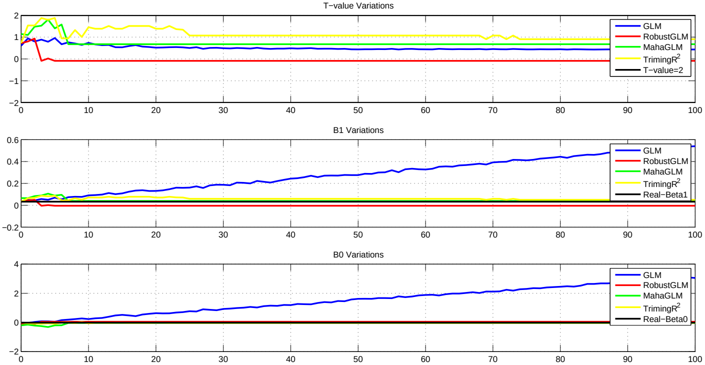

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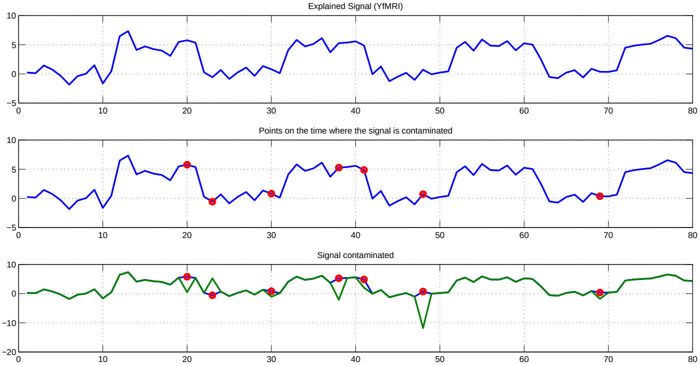

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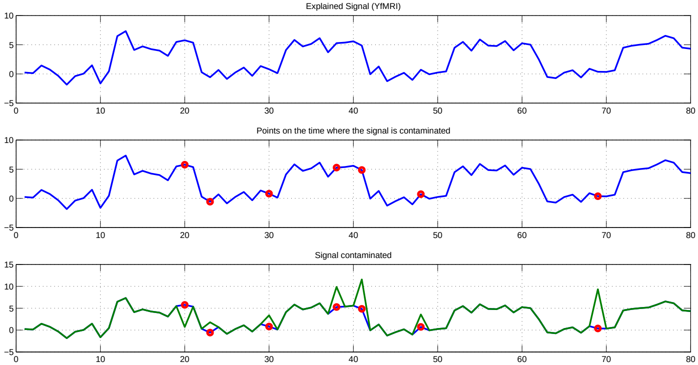

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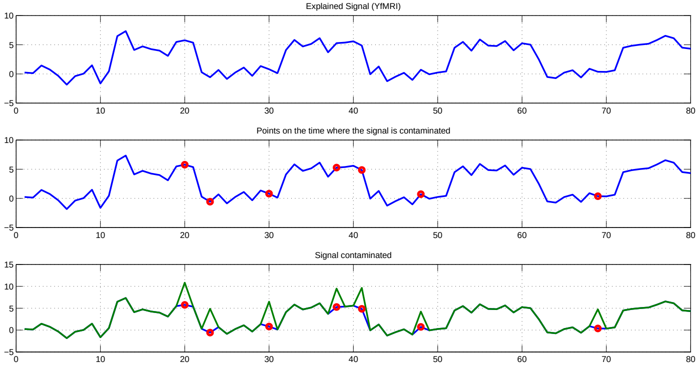

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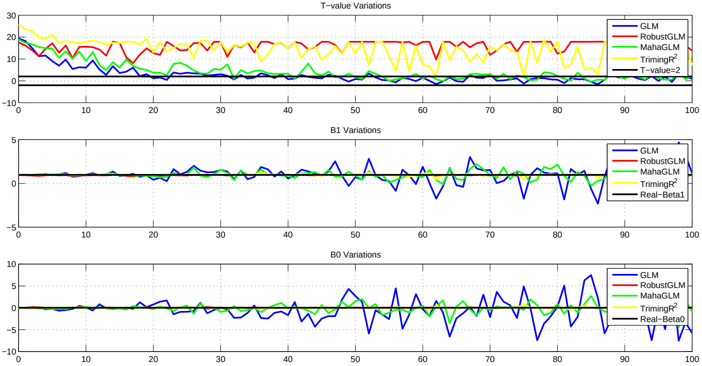

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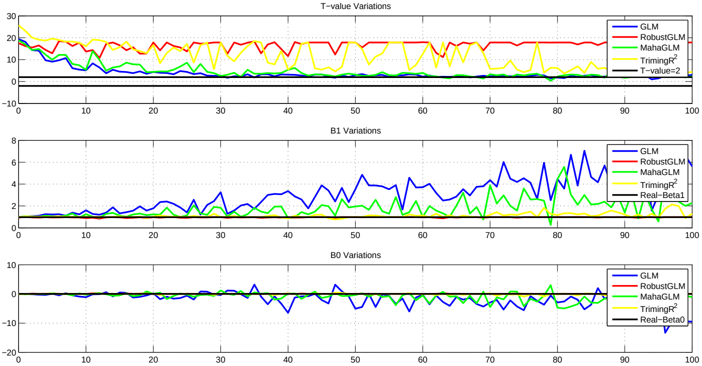

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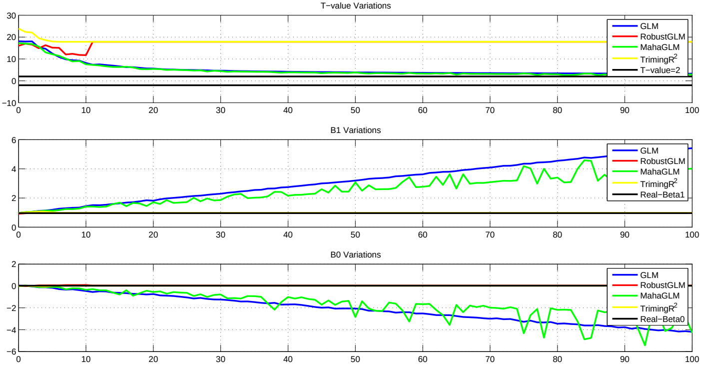

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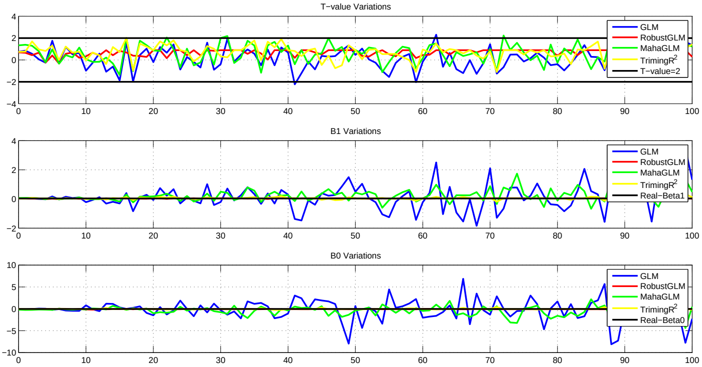

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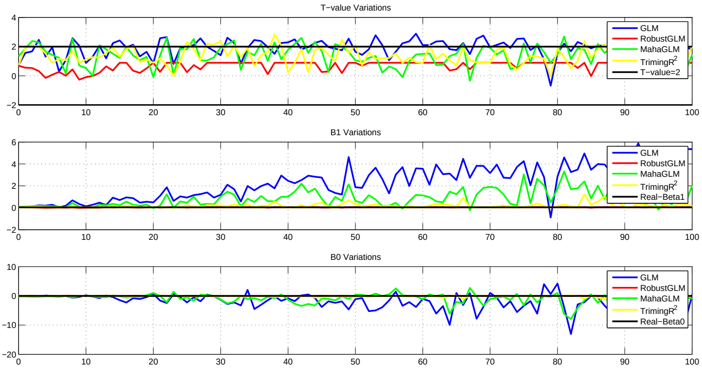

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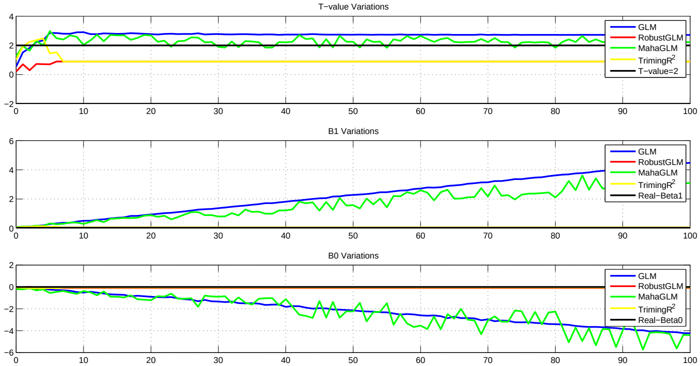

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Non-parametric estimation for Σ

Pearson correlation coefficient

The Pearson correlation matrix is related to the covariance matrix by

$$\rho_p(i,j) = \frac{C(i,j)}{\sqrt{C(i,i)C(j,j)}} \tag{1}$$

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- **Robust version** $\rho_r(i,j)$

Kendall

$$\rho_k(i,j) = \frac{2 * (N_c - N_d)}{N(N-1)}$$

Spearman

$$\rho_s(i,j) = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

Robust version

$$\rho_r(i,j) = \frac{C_r(i,j)}{\sqrt{C_r(i,i) C_r(j,j)}}$$

where,

$$C_r(i,j) = \text{median}\{(x_i - \text{median}(x_i))(x_i - \text{median}(x_i))\}$$

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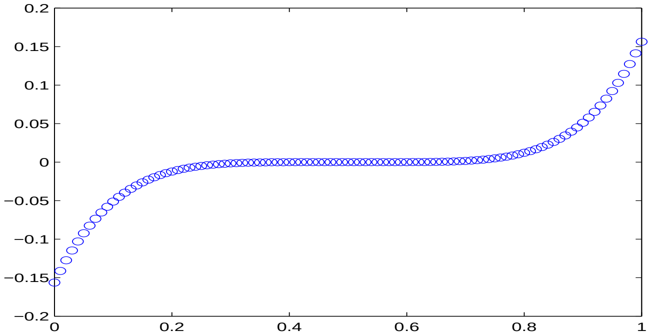

Figure: $\rho_p = 0.81, \quad \rho_k = \rho_s = \rho_r = 1$

Non-parametric estimation for Σ

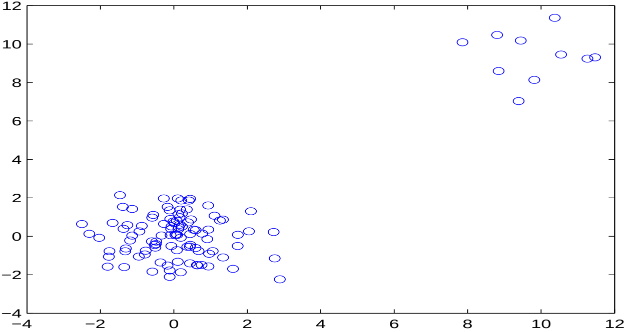

Figure: $\rho_p = 0.86, \quad \rho_k = 0.15, \quad \rho_s = 0.23, \quad \rho_r = 0.005$

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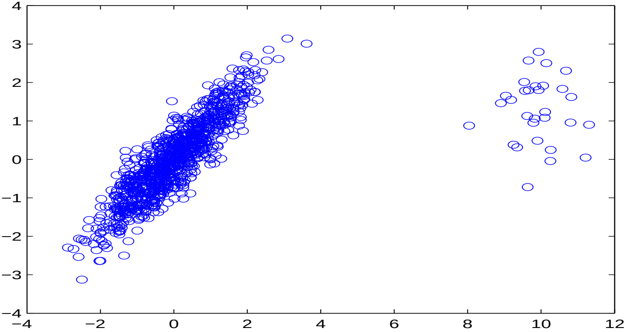

Figure: $\rho_p = 0.61, \quad \rho_k = 0.77, \quad \rho_s = 0.89, \quad \rho_r = 0.83$

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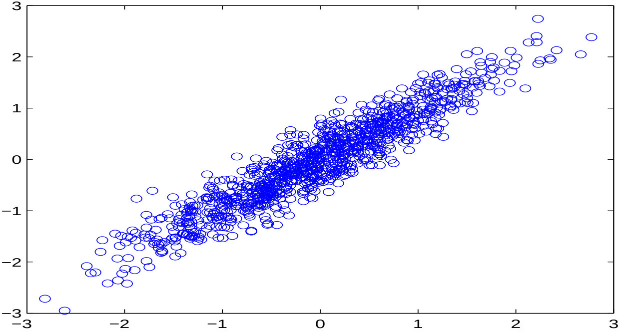

Figure: $\rho_p = 0.94, \quad \rho_k = 0.85, \quad \rho_s = 0.93, \quad \rho_r = 0.92$

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- **The estimations introduced here improve the performance of GLM**

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